Possibility, Impossibility, and History in the Origins of the Marriage Tax

Abstract - *We examine the marriage tax in a formal setting. We begin with precise statements of the basic normative principles in the literature and proofs of old and new impossibility claims. Along the way we document and resolve important ambiguities in the existing literature. We then characterize the tax systems that are piecewise linear, linear, and proportional in terms of familiar normative principles and a new principle we call local marriage neutrality. Earlier analytical work emphasizes that all tax structures must violate certain principles. Local marriage neutrality permits a more nuanced analysis of the conflicts among tax principles that is more relevant to the study of moderate tax reform. We use these results to give a brief yet comprehensive analysis of the properties of the income tax system at key points in time. The last section uses the same results to offer observations about recently proposed and enacted changes for reducing the marriage tax.*

INTRODUCTION

A tax system contains a marriage tax if it is possible for a married couple to owe more in tax than their total liability would be as two unmarried individuals. It has been a basic feature of the U.S. federal personal income tax code since 1969. There are many analyses of the income tax and the marriage tax, such as those by Bittker (1975), Munnell (1980), O'Neill (1983), Fraser (1986), CBO (1997), Bartlett (1998), Alm, Dickert–Conlin, and Whittington (1999), and Steuerle (1999), to name just a few. All emphasize that certain normative principles for a tax system are incompatible, that this creates a need for tradeoffs, and that the marriage tax results from a particular tradeoff. Only the analysis by Fraser (1986) is formal, however, and it has gone largely unnoticed in the larger literature on taxation. This casual approach to important theoretical questions has led to some conflation of distinct normative principles, ambiguous statements about the tradeoffs they imply, inaccuracies in analysis and communication, and a general absence of accumulated wisdom about effective analytical approaches.

We begin by formalizing the principles in the literature and presenting canonical impossibility results. One of these results is fairly well–known and proved in Fraser (1986). We show that it has been confused with other impossibility results and with one impossibility claim that is not, in fact,

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correct. These and related problems with the literature are not well–known and we take some care in documenting them.

We then focus on two principles that have played different roles in the history of the income tax but have not been clearly delineated and analyzed. The first is the principle of equal treatment of married couples, which states that all married couples with the same total income pay the same tax. The second is the stronger principle of equal payments by singles and couples, which states that a married couple pays the same tax that a single individual with that income would pay. We show that if a tax system satisfies marriage neutrality (no marriage taxes or bonuses), equal *treatment* of married couples, and a mild continuity condition, then the tax system must be linear. A linear tax can be progressive, so progressivity is consistent with marriage neutrality and equal treatment. This result is mentioned in Fraser (1986) and the mathematical literature on functional equations but does not appear in the literature on taxation. We then show that only a proportional tax system can satisfy marriage neutrality and equal *payments*. Equal payments is therefore substantively more restrictive than equal treatment because proportionality rules out progressivity while linearity does not.

These first two results have some implications for tax policy, but we use them mostly to highlight difficulties with the existing tax literature. Our main result is directed towards policy. We explore a weaker condition than marriage neutrality that we call *local* marriage neutrality. This requires marriage neutrality for just those couples whose individual members have similar incomes. It is consistent with increasing marginal tax rates and marriage bonuses. Recent proposals to eliminate the marriage tax have really been proposals to achieve neutrality for people with similar incomes who marry while preserving bonuses for all others, and local marriage neutrality is consistent with these goals.

Our main theoretical result is that, given a mild continuity condition, local marriage neutrality, equal treatment, and a general marriage bonus are equivalent to requiring the tax system to have a specific structure. This is useful for policy purposes because it means that there is no other tax structure, just waiting to be discovered, that will meet these criteria. Put somewhat differently, it is no coincidence that recent proposals to eliminate the marriage tax but preserve some marriage bonuses and keep equal treatment all look roughly the same.

Our theoretical results allow us to give a brief, comprehensive, and precise statement of the properties that the tax system satisfied and the tradeoffs that were present at key points in time, up to the emergence of the marriage tax in 1969. This historical material is "known" to marriage tax aficionados, but our formal treatment provides a stronger basis for this knowledge and the ability to communicate the results clearly. These results are also useful for understanding the evolution of the income tax, but a complete analysis of this material would require a detailed political–economic model that is outside the scope of this analysis.

Finally, we discuss proposals to reduce the marriage tax. Our initial results are useful if large–scale reform is under consideration. Our main result is useful for evaluating more modest, and therefore in some ways more relevant, reforms, like those in the Economic Growth and Tax Relief Reconciliation Act of 2001. Earlier analytical work emphasizes that all tax structures must violate certain strong principles. Weakening marriage neutrality to local marriage neutrality permits a more nuanced analysis of the conflicts among tax principles that is more relevant to (relatively) moderate tax reform.

The next section presents the formalization of tax principles and all of the theorems. The third section presents the his-

torical analysis, and the fourth section considers the issue of reducing the marriage tax. The fifth section concludes. All proofs but the first are contained in an appendix.

FRAMEWORK AND MAIN RESULTS

Every individual is either single or married and has a non–negative level of taxable income. The tax liability of a single individual depends only on his or her income and is given by a function $T^s: R_+ \to R$, called *the tax schedule for single individuals*. Note that at the moment we place no restrictions whatsoever on T^s. It may take any value at zero income and it may increase, decrease or jump discontinuously with income. The tax liability of a married couple depends only on the taxable incomes of both individuals and is given by some function $T^m: R_+^2 \to R$, called *the tax schedule for married couples*. Any pair (T^s, T^m) is a *tax system*.[1]

A tax system satisfies *marriage neutrality* if for all incomes the tax liability of a married couple equals the sum of the payments each individual would make if single. Formally, for all $y \geq 0$ and $y' \geq 0$:

$$T^m(y, y') = T^s(y) + T^s(y').$$

The tax system contains a *marriage bonus* if there exist incomes y and y' such that $T^m(y, y') < T^s(y) + T^s(y')$ and a *marriage tax* if $T^m(y, y') > T^s(y) + T^s(y')$. A tax system contains a *general marriage bonus* if $T^m(y, y') \leq T^s(y) + T^s(y')$ for all incomes with strict inequality for some, and a *general marriage* tax if $T^m(y, y') \geq T^s(y) + T^s(y')$ for all incomes with strict inequality for some.

A tax system satisfies *equal treatment of married couples* (or just *equal treatment*) if all married couples with the same total

income have the same tax liability. Formally, there must exist a function $t^m: R_+ \to R$ such that for all incomes:

$$T^m(y, y') = t^m(y + y').$$

A stronger condition is *equal payments by singles and couples* (or just *equal payments*), which holds if for all $y \geq 0$ and $y' \geq 0$:

$$T^m(y, y') = T^s(y + y').$$

This says that the tax schedule for married couples is *identical* to that for singles. A married couple pays the same tax on total income that a single individual would pay on that income. We say that a tax system *favors married couples over single individuals* (or just *favors married couples*) if $T^m(y, y') \leq T^s(y + y')$ for all incomes with strict inequality for some. Finally, a tax system has *income splitting* if for all incomes:

$$T^m(y, y') = 2T^s\left(\frac{y + y'}{2}\right).$$

This is an historically important tax structure that provides an interesting compromise among certain tax principles. We return to it in Theorem 3 below.

A central theme of our analysis is the importance of distinguishing equal payments from both equal treatment and marriage neutrality.[2] It is well known that equal payments has played a role in the political evolution of the tax code that is distinct from that of the other two conditions. We discuss this further in the third section. It is less well understood that equal payments is significantly stronger than equal treatment and implies more severe tradeoffs. Our theorems below make this precise. These results also show that equal payments is quite distinct from marriage neutrality. Combining them into

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[1] "Tax liability" here is just the tax directly implied by taxable income and marital status. We assume, along with all other analytical (as opposed to computational) treatments of the marriage tax, that the tax principles we examine are normative relative to these concepts of income and liability.

[2] A careful and critical analysis of equal payments as a distinct normative principle is in Bittker (1975, pp. 1419–26). A more recent analysis is in Alm, Whittington, and Fletcher (2002).

a single "neutrality" condition, as is done in some analyses, would block any examination of how they trade off against each other and against other principles.[3]

The tax schedule T^s is *progressive* if the average tax rate strictly increases with income. Formally, for any $y > 0$ and $y' > 0$:

$$\frac{T^s(y + y')}{y + y'} > \frac{T^s(y)}{y}.$$

It has *increasing marginal rates* if, given any two income levels, a common increase generates the same or greater increase in tax liability from the higher income, and there are income levels for which a common increase generates a strictly greater increase in tax liability from the higher income. Formally, for all income levels $y_2 \geq y_1 \geq 0$ and all $c \geq 0$ we have $T^s(y_1 + c) - T^s(y_1) \leq T^s(y_2 + c) - T^s(y_2)$, with strict inequality for some y_1, y_2, and c. Note that a linear tax schedule with a positive transfer at zero income is progressive but has a fixed marginal rate. This case is important in the analysis below. More generally, the tax schedule has *changing marginal rates* if there exist two income levels $y_1 \geq 0$ and $y_2 \geq 0$ and a number $c > 0$ such that $T^s(y_1 + c) - T^s(y_1) \neq T^s(y_2 + c) - T^s(y_2)$.[4]

In order to illustrate these concepts and some of our results, consider the following linear tax system:

$$T^s(y) = a_s + b_s(y)$$

$$T^m(y + y') = a_m + b_m(y + y').$$

Progressivity requires $a_s + b_s(y_1 + y_2) / y_1 + y_2 > a_s + b_s(y_1)/y_1$ for all positive incomes. After rearranging this gives $a_s y_2 < 0$, so $a_s < 0$. Marriage neutrality requires $T^m(y_1 + y_2) = T^s(y_1) + T^s(y_2) = 2a_s + b_s(y_1 + y_2)$, so necessarily $a_m = 2a_s$ and $b_m = b_s$. Equal treatment holds without any further restrictions, so it is consistent with progressivity and marriage neutrality. In contrast, equal payments requires $T^m(y_1 + y_2) = T^s(y_1 + y_2) = a_s + b_s(y_1 + y_2)$, so $a_m = a_s$ and $b_m = b_s$. If we require equal payments and marriage neutrality then we must have $b_m = b_s$ and both $a_m = 2a_s$ and $a_m = a_s$, which forces $a_s = 0$. Equal payments and marriage neutrality are therefore not consistent with progressivity. In Theorem 2, we show that this result essentially holds even without the seemingly restrictive assumption of linearity.

In fact, we can say a bit more. Progressivity (which gives a_s negative) and equal payments imply a general marriage tax, since in this case $T^m(y_1 + y_2) > 2a_s + b_s(y_1 + y_2) = T^s(y_1) + T^s(y_2)$. In contrast, progressivity and marriage neutrality imply the tax system favors married couples over single individuals, since in this case $T^m(y_1 + y_2) = T^s(y_1) + T^s(y_2) = 2a_s + b_s(y_1 + y_2) < T^s(y_1 + y_2)$. Again, these results hold even without linearity (Theorem 1).

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[3] Munnell (1980) combines the two principles in her discussion of the 1969 Tax Reform Act when stating, "The law, however, violates the principle of marriage neutrality. A single person generally pays more tax than a married couple with the same income; and a married couple may pay more or less than they would if they were single (p. 259)." CBO (1997) seems to combine the two principles inadvertently in some discussions. They define marriage neutrality as we do. However, they state, "The 1948 'solution'… imposed higher taxes on single people in relation to married couples, thus violating the principle of marriage neutrality (p. 8)." The statement describes a violation of equal payments, not marriage neutrality. In discussing the 1948 reform, O'Neill (1983) says, "From the point of view of the single person contemplating marriage, the 1948 tax act imposed a 'marriage bonus' or 'single's penalty' since by marrying someone with little or no income, a person could substantially lower his tax burden (p. 4)." The expressed difference, which is between a single person's tax and a married couple's tax, represents a violation of equal payments (but see the discussion after Theorem 1).

[4] It is the fact that the rates are changing, not increasing per se, that is at the heart of the first impossibility result below. Note that while marginal tax rates generally increase with income, the Tax Reform Act of 1986 essentially created a 28 percent bracket above the 33 percent bracket (Auerbach and Slemrod, 1997). It is straightforward to show that changing marginal rates is equivalent to the condition Fraser (1986) calls, "nonlinearity for single taxes."

Our first theorem presents key impossibility results. We state the results and then link them back to claims in the existing literature. The proofs are completely algebraic, in contrast to those of the possibility theorems further on.

Theorem 1

Given a tax system (T^s, T^m),

a. If T^s has changing marginal rates, then the tax system cannot satisfy both marriage neutrality and equal treatment of married couples.
b. If T^s is progressive, then the tax system cannot satisfy both marriage neutrality and equal payments by singles and couples.
c. More specifically, if T^s is progressive and $T^s(0) \leq 0$:
   (i) if the tax system satisfies equal payments by singles and couples, then it has a general marriage tax.
   (ii) if the tax system satisfies marriage neutrality, then it favors married couples over single individuals.

Proof

a. Suppose not, so all three properties hold. Fix any $y \geq 0$ and $y' \geq 0$. Marriage neutrality and equal treatment imply:

[1] $T^s(y) + T^s(y') = T^m(y, y')$
$= t^m(y + y') = T^m(y + y', 0)$
$= T^s(y + y') + T^s(0)$.

Therefore:

[2] $T^s(y + y') = T^s(y) + T^s(y') - T^s(0)$,
for $y \geq 0, y' \geq 0$.

Now fix any $y_1 \geq 0$, $y_2 \geq 0$, and $c > 0$. Equation [2] implies:

$T^s(y_1 + c) - T^s(y_1) = T^s(c) - T^s(0)$
$= T^s(y_2 + c) - T^s(y_2)$.

This contradicts changing marginal rates.

b. From the definition of progressivity, if $y > 0$ and $y' > 0$ then $y\,(T^s(y + y')/y + y') > T^s(y)$ and $y'(T^s(y + y')/y + y') > T^s(y')$. Adding both sides and factoring gives:

[3] $T^s(y + y') > T^s(y) + T^s(y')$,
for $y > 0, y' > 0$

For all incomes, equal payments and marriage neutrality give $T^s(y + y') = T^m(y, y') = T^s(y) + T^s(y')$, respectively. This contradicts [3].

c. Given $T^s(0) \leq 0$, $y' = 0$ implies $T^s(y + y') = T^s(y) \geq T^s(y) + T^s(y')$. The same holds if $y = 0$ or both are zero. Progressivity gives [3], so we have for all incomes $T^s(y + y') \geq T^s(y) + T^s(y')$. Now:

   (i) if the tax system satisfies equal payments, then for all incomes $T^m(y, y') = T^s(y + y') \geq T^s(y) + T^s(y')$ with strict inequality for positive incomes. There is a general marriage tax.
   (ii) if the tax system satisfies marriage neutrality, then for all incomes $T^s(y + y') \geq T^s(y) + T^s(y') = T^m(y, y')$ with strict inequality for positive incomes. The tax system favors married couples. ‖

Theorem 1(a) is stated in Fraser (1986), Alm and Whittington (1997), and Steuerle (1999) and proved in Fraser. The remainder of Theorem 1 is new.

One claim we do not make is that equal treatment, marriage neutrality and progressivity are inconsistent. We have already seen that this claim is false. Demonstrations that it is true appear throughout the literature on the marriage tax, how-

ever. These demonstrations are essentially of two types. One type asserts that the three conditions are inconsistent but uses increasing marginal rates in a numerical example (Bittker, 1975; Rosen, 1977; and Bartlett, 1998). Setting aside questions about how progressivity is defined,[5] these examples could at best demonstrate that *some* tax system violates equal treatment, marriage neutrality and increasing marginal rates. Theorem 1(a) shows that *all* tax systems violate these conditions.

The other type of demonstration is analytical, not numerical, but adds an additional condition to the definition of marriage neutrality (Munnell, 1980; and Graetz, 1997; CBO, 1997 is slightly different as explained below). This condition is that marrying an individual with no income does not change one's tax liability. Formally, given any $y > 0$, the tax system must satisfy $T^m(y, 0) = T^s(y)$. The inconsistency is then easy to show. Given $y > 0$ and $y' > 0$, this additional condition with equal treatment and marriage neutrality respectively imply $T^s(y + y') = T^m(y + y', 0) = T^m(y, y') = T^s(y) + T^s(y')$. This violates progressivity (equation [3]).[6]

While this demonstration is fine, marriage neutrality is now generally defined without requiring $T^m(y, 0) = T^s(y)$. Without this condition there is no inconsistency created by equal treatment, marriage neu-

trality, and progressivity. The changing definition of marriage neutrality seems to have created some confusion about its implications in the tax literature.

We now focus on the difference between equal treatment and equal payments in the presence of marriage neutrality. Earlier we presented a linear tax system satisfying marriage neutrality and equal treatment (it required $a_m = 2a_s$). We now show that if the tax schedule for single individuals satisfies a mild continuity condition—it must be continuous at one or more points—and the tax system satisfies marriage neutrality and equal treatment, then the tax system is the same as the one in the example. Similarly, we presented a linear tax system satisfying marriage neutrality and equal payments (it required $a_m = a_s = 0$). We now show that these principles (with the continuity condition) imply that the tax system is the same as the one in the example. Thus, equal payments is substantively more restrictive than equal treatment because the former rules out progressivity while the latter does not.[7]

Theorem 2

Assume that a tax system satisfies marriage neutrality and equal treatment of married couples with T^s continuous at one or more points.

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[5] Rosen uses our definition, Bartlett provides no definition, and Bittker refers to "progressive rate" taxation.

[6] The proof in Graetz (1997, pp. 294–95) comes from testimony delivered in 1972 by an assistant secretary of the Treasury before the House Ways and Means Committee. The role of the additional condition is clear there. The proof in Munnell (1980, p. 249) is represented in a chart that comes from a 1977 report by the U.S. Treasury, Office of Tax Analysis, "Federal Income Tax Treatment of Married and Single Taxpayers" (the chart is also reproduced in Fraser (1986, p. 832)). The chart summarizes the same examples given in the testimony. "Marriage neutrality" appears twice on the chart, once to establish $T^m(y, y') = T^s(y) + T^s(y')$ and once to establish $T^m(y, 0) = T^s(y)$.

   In an interesting twist, CBO (1997, p. 3) cites the same chart but modifies it. Given $y > 0$ and $y' > 0$, they use marriage neutrality (as we define it) and equal treatment to establish $T^s(y + y') + T^s(0) = T^m(y + y', 0) = T^m(y, y') = T^s(y) + T^s(y')$. They then state that this violates progressivity. This is not true if $T^s(0) < 0$, so this does not establish that the principles conflict.

[7] The fact that marriage neutrality and equal treatment imply a linear tax structure is almost entirely unknown in the literature on the marriage tax. We developed our own proof before learning that the proposition is stated without proof in Fraser (1986), p. 839, and is a straightforward implication of a result (Lemma 8.3.2) in Castillo and Ruiz–Cobo (1992). We thank Dave Heulse and Claudio Zoli for drawing our attention to this work, respectively. The converse, that a linear tax structure has these properties, is well–known. See for example CBO (1997), p. 56, and Steuerle (1999), p. 39, who also notes that "flat" taxes may be progressive.

a. It is equivalent to assume that there are real numbers a and b such that the tax schedules take the linear form:

[4] $T^s(y) = a + by$

[5] $T^m(y, y') = 2a + b(y + y')$.

If we add progressivity to the premise, then it is equivalent to assume that the tax system takes this form with $a < 0$.

b. If we replace "equal treatment" with "equal payments" in the premise, then it is equivalent to assume that there is a real number b such that the tax schedules take the form $T^s(y) = by$ and $T^m(y, y') = b(y + y')$.

Proof

All remaining proofs are in the appendix.

Thus, in the presence of marriage neutrality (and the weak continuity condition), equal payments forces the vertical intercept of the implied tax schedule for single individuals to pass through the origin. If marriage neutrality and equal payments were the only desirable principles, then this tax structure would be the only possible choice.

Our final theorem examines a weaker condition than marriage neutrality that we call *local marriage neutrality*. This condition requires marriage neutrality for just those couples whose individual members have similar incomes. Our interest in this condition is pragmatic. First, local marriage neutrality is consistent with increasing marginal tax rates while marriage neutrality is not (Theorem 1(a)). Increasing marginal rates appears to be an inevitable part of the tax code, so it is useful to examine conditions that can be combined with it. Second, recent proposals to

eliminate the marriage tax have really been proposals to achieve neutrality for people with similar incomes who marry while preserving bonuses for those with diffierent incomes. This suggests that only local marriage neutrality is of concern to policymakers, if only implicitly. Finally, all of these recent proposals move the tax system closer to one in which the tax brackets for married couples are twice as large as those for single individuals, the tax rates for couples and singles are the same on corresponding income brackets, and the tax rates increase with the income brackets.[8] We say that any tax system with these properties has the "Double Bracket-Increasing Common Rate" (DB-ICR) structure. Our main result below shows that, given a mild continuity condition, equal treatment, local marriage neutrality and a general marriage bonus are equivalent to requiring the tax system to have the DB-ICR structure.

Formally, fix $z_0 = 0$ and $z_i > z_{i-1}$ for $i = 1$, $2, \ldots$, where $\lim_{i \to \infty} z_i = \infty$. We refer to the closed interval $[z_{i-1}, z_i]$ as "interval i." A tax system (T^s, T^m) satisfies *local marriage neutrality* if for all i and for all incomes y and y' in interval i we have $T^m(y, y') = T^s(y) + T^s(y')$.

Theorem 3

Assume that a tax system satisfies local marriage neutrality and equal treatment of married couples with T^s continuous at one or more points in each interval.

a. It is equivalent to assume that for each interval i there are real numbers a_{i-1}, a_i and b_{i-1}, b_i such that $a_{i-1} + b_{i-1}z_i = a_i + b_iz_i$ and the tax schedules take the piecewise linear and continuous form:

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[8] For example, Senate Bill S11 introduced on January 22, 2001, read in part, "To amend the Internal Revenue Code of 1986 to eliminate the marriage penalty by providing that the income tax rate bracket amounts, and the amount of the standard deduction, for joint returns shall be twice the amounts applicable to unmarried individuals."

$$T^s(y) = \begin{cases} a_0 + b_0 y & \text{for} \quad y \in [z_0, z_1] \\ a_1 + b_1 y & \text{for} \quad y \in [z_1, z_2] \\ \vdots \end{cases}$$

$$T^m(y, y') = t^m(y + y') = \begin{cases} 2a_0 + b_0(y + y') & \text{for} \quad y + y' \in [2z_0, 2z_1] \\ 2a_1 + b_1(y + y') & \text{for} \quad y + y' \in [2z_1, 2z_2] \\ \vdots \end{cases}$$

b. If we add to the premise that the tax system contains a general marriage bonus, then it is equivalent to assume that the tax system has the form in (a) and $b_{i-1} < b_i$ for all $i \geq 1$ (i.e., it has the DB-ICR structure).

c. If the tax system has the form in (a), $b_{i-1} < b_i$ for all $i \geq 1$, and $a_0 \leq 0$, then it satisfies income splitting, it contains a general marriage bonus, and it favors married couples over single individuals. If instead only T^s has the form in (a) while $T^m(y, y') = T^s(y + y')$ (so equal payments holds), $b_{i-1} < b_i$ for all $i \geq 1$, and $a_0 \leq 0$, then it contains a general marriage tax.[9]

Part (b) of Theorem 3 is especially useful for policy purposes because it shows that if a tax system satisfies local marriage neutrality and equal treatment and has a general marriage bonus then it must have the DB-ICR structure. There is no other tax structure, just waiting to be discovered, that could give a tax system with these properties.

Part (c) brings together all of the key properties of the DB-ICR structure under a very mild additional requirement ($a_0 \leq 0$). We use it to provide brief, comprehensive and precise statements of the properties that the tax system satisfied and the tradeoffs that were present at key points in time, up to the emergence of the marriage tax in 1969.

AN ANALYTICAL HISTORY OF THE INCOME TAX AND THE MARRIAGE TAX

The ratification of the sixteenth amendment in 1913 cleared the way for the early income tax statutes. By 1931, various Supreme Court and Internal Revenue Service decisions led to the following situation. In community property states the tax system had the DB-ICR structure. More specifically, the tax schedule for single individuals had the structure in Theorem 3(a) and the tax rates increased with income brackets. The tax schedule for married couples was defined by income splitting. Marriage vested in each spouse ownership of half of all income from virtually all sources.[10] Thus, if y and y' were the taxable incomes of an unmarried couple, each had a taxable income of $y + y'/2$ as a married couple and their total liability was $2T^s(y + y'/2)$. The tax schedule for single individuals and income splitting give the DB-ICR structure. There was zero tax liability on zero (taxable) income, so Theorem 3(c) also holds. There was both a general marriage bonus and a generally lower tax payment for married couples than for a single individual with the same income.

In contrast, in common law property states, salary and property income were the taxable income of the recipients regardless of marital status.[11] Thus, in common law property states, the tax liability for the couple was $T^m(y, y') = T^s(y) + T^s(y')$.

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[9] Theorem 3(c) does not mention progressivity. If the tax system has the form in (a), $b_{i-1} < b_i$ for all $i \geq 1$ and $a_0 < 0$ then the tax system is progressive (we prove this as part of Theorem 3(c)). When $a_0 = 0$ progressivity fails because the average tax rate is merely constant on the first interval and not strictly increasing. This is an artifact of the standard definition of progressivity.

[10] Poe v. Seaborn, 282 U.S. 101 (1930); Bittker (1975), footnote 32. In 1930 the recognized community property states were Arizona, Idaho, Louisiana, Nevada, New Mexico, Texas, and Washington; Bittker, pp. 1407–08. California was added in 1931; Bittker, fn. 44.

[11] Lucas v. Earl, 281 U.S. 111 (1930); Bittker (1975, p. 1401).

The tax system therefore satisfied marriage neutrality.[12] Since there were changing marginal rates, it necessarily failed equal treatment (Theorem 1(a)) and by implication equal payments.

In effect, one federal tax statute created two distinct tax systems based on geography. Furthermore, the marriage bonus in community property states meant that the tax liability for a married couple was lower than it was for an identical couple in a common law property state. This systematic difference led to a scramble in which a number of common law property states ultimately enacted community property systems.

In 1941, the House Ways and Means Committee considered equal payments as a way to remove differences in tax liability based on geography. Those differences stemmed from the computation of a husband and wife's individual incomes, and those are irrelevant in any system satisfying equal treatment. Equal payments would have created a general marriage tax (Theorem 3(c)), however, and it would have increased the tax liability of married couples in all states (*ceteris paribus*). These properties were well understood at the time.[13]

Ultimately, Congress went in the other direction. In 1948 it introduced a joint return for which the brackets were double the size of those on the individual return. This effectively permitted income splitting everywhere. The federal income tax system then inherited all of the properties established above for the tax systems in community property states: local mar-

riage neutrality, equal treatment, a general marriage bonus (all from the DB-ICR structure), and favoring married couples.

By 1969, single individuals were paying as much as 40 percent more than couples with the same income.[14] This was true despite the fact that in the early 1950s Congress reduced the taxes on single individuals who were also heads of households, the rationale being that some of the tax benefits of marriage were an allowance for family responsibilities.[15] The remedy Congress found in 1969 was to disconnect the tax schedule for married couples from the tax schedule for single individuals. The tax brackets for single individuals remained half the size of those for married couples, but the tax rates were lower.[16] We know immediately that local neutrality could no longer hold (Theorem 3(a)). Individuals with identical incomes who married would pay a marriage tax since the wider brackets would provide no advantage and they faced higher rates. Individuals with very different incomes who married would still receive a marriage bonus, however, because the advantage of the wider brackets would more than offset the effect of the higher rates.

REDUCING THE MARRIAGE TAX

Our analysis focuses entirely on tax structure. We explore the logical possibilities of marriage bonuses, taxes, and neutrality, and our results can provide some guidance for reducing marriage taxes. It is important to note, however, that the magnitudes of bonuses and taxes depend on

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[12] This should not be taken too literally. In 1913, two unmarried individuals could each claim a personal exemption of $2,500, but a married couple could exempt at most $3,333 (Foster, 1914, p. 192j) whether they chose to file two returns or one. Thus, the total liability of the married couple was given by the sum of their separate liabilities on taxable income, but these separate liabilities were not necessarily the same as their respective liabilities if single because their respective taxable incomes could change.

[13] Bittker (1975, pp. 1408–11). A couple's tax liability would tend to rise because $2T^s[(y + y')/2] \leq T^s(y) + T^s(y') \leq T^s(y + y')$ for all incomes with strict inequality for some (this is shown in proving the first part of Theorem 3(c)).

[14] Rosen (1977, p. 424).

[15] Bittker (1975, p. 1417). Our model does not incorporate this category, but it could be extended to do so.

[16] Tax Foundation (2000, p. 122).

"who is marrying whom" and how much income they earn. This demographic information, along with changes in tax law, are essential for a full picture of how marriage taxes and bonuses have changed over time as well as for determining the quantitative effect of any particular change in the law.[17]

Furthermore, we have made no attempt to capture the effects of various assistance programs like welfare and the Earned Income Tax Credit. A marriage bonus is likely if two individuals marry who have unequal incomes and neither of them participate in any assistance program. A marriage tax, however, is possible under the same circumstances for individuals who receive assistance, because transfer programs themselves contain marriage penalties. More generally, the penalties in the transfer programs may be offset or reinforced by the properties of the tax code.[18] If we define the tax code to include the transfer programs, then progressivity and increasing marginal rates are not even stylized facts for anyone but middle and upper–income individuals.[19]

We have shown that a linear tax is the only way to achieve progressivity, marriage neutrality and equal treatment (Theorem 2(a)). Proposals to reform the tax system so it better meets these goals must, in one way or another, move the tax system towards this structure. They must reduce the impact of increasing marginal rates on tax liability and harmonize the liabilities of single individuals and married couples. This is the overarching logic that drives suggestions like those in CBO (1997) and Steuerle (1999),

who propose widening the tax brackets, reducing the range of tax rates, increasing the standard deduction for joint filers, and using the direct tax rate schedule to establish progressivity rather than phase–outs in the assistance programs.

Marriage neutrality in itself could be achieved by returning to individual filing as it existed in the common law property states before 1948. This would also end the equal treatment of married couples, of course, and this is the reason proposals of this sort are usually rejected.[20] Congress would also have to craft careful legislation to avoid recreating differences in tax liability based on geography. A key point, however, is that the closer the tax system comes to that described in Theorem 2(a), the more irrelevant is the distinction between joint versus individual filing. Under that system, the tax liability of a married couple is the same whether it is computed as the sum of the individual liabilities under individual filing or as the liability from joint filing computed from the tax schedule for married couples.[21]

We also show that a tax system with the DB-ICR structure is the only way to achieve local marriage neutrality, equal treatment and a general marriage bonus. There is no other tax structure, just waiting to be discovered, that could give a tax system with these properties. This is the overarching logic behind the marriage penalty relief in the proposal cited prior to Theorem 3 and in the Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA).[22] Of

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[17] Eissa and Hoynes (2000) and CBO (1997).

[18] A recent discussion and analysis of these interactions is in Dickert–Conlin (1999).

[19] This point is essentially made by Steuerle (1999), p. 40.

[20] Bittker (1975, pp. 1437–39); CBO (1997, pp. 54–5).

[21] Steuerle (1999) observes that individual filing with linear taxes and a positive transfer would mean that the nonworking spouse in a wealthy couple would be entitled to welfare benefits. He then states, "worrying about whether someone earning millions gets a small subsidy here or there is simply not worth the trouble. On average, higher income families can be made to pay for these changes through an explicit tax rate structure (p. 41)."

[22] See footnote 8. EGTRRA provides that the standard deduction for married couples will be exactly double that for single individuals by 2009; the first (10%) bracket for married couples is double that for single individuals immediately; and the second (15%) bracket for married couples will be double that for single individuals by 2008. Single individuals with both incomes in the same bracket who marry will see no bonus or penalty, and those with different incomes (but in these brackets) who marry will see a bonus. House Bill 2 (now Public Law 108-27) accelerates these provisions.

course, these proposals imply a system that favors married couples over single individuals (Theorem 3(c)).

We close with a proposal that is suggested by these results and merits further research. Equal treatment and local marriage neutrality require the tax system to have the form in Theorem 3(a). It can have this structure and progressivity, however, without having increasing marginal tax rates. If the marginal rate for any bracket i is less than the marginal rate for bracket $i - 1$ yet larger than the *average* rate at the upper bound of that bracket, then the tax structure can still be progressive.[23] Unlike the current tax system, this system would be marriage neutral for couples whose individual members have similar incomes. Since it must contain a marriage tax (Theorem 3(b)), however, some couples with different incomes would see their total tax liability increase. Beyond this, it is an open question whether its treatment of married couples, unmarried couples and single individuals is more appealing than under both the current system and reform proposals. The implications for efficiency would also be of interest.

CONCLUSION

One theme of this analysis has been the importance of making two distinctions. The first is between equal payments by singles and couples and equal treatment of married couples. Equal payments is significantly stronger than equal treatment in the sense that the former implies a tradeoff between marriage neutrality and progressivity while the latter does not. The second distinction we emphasize is between increasing marginal tax rates and progressivity. Increasing (or more generally changing) marginal tax rates is stronger than progressivity in the sense that it implies a tradeoff between marriage neutrality and equal treatment. Progressivity

only implies a tradeoff between marriage neutrality and equal payments. It is fair to say that most of the literature on the income tax and marriage tax has not made these distinctions clear, a fact we take some care to document at the start.

A second theme of this analysis has been the value for policy purposes of deriving the tax structures that are implied by tax principles. We do this in our second and third theorems. This analysis is useful for policy purposes because it means that there are no other tax structures, just waiting to be discovered, that could give tax systems with the desired properties. The existing applied literature seems to take this idea for granted, but that is only because there is some confusion between this idea and its converse, that if a tax system has a certain form then it obeys certain principles.

One important implication of our main result is that *all* proposals to eliminate the marriage tax while preserving marriage bonuses for some couples and equal treatment of married couples must look roughly the same. Another implication is that one can track the deviations from principles that will result from deviations in the required tax structure. In this light, the marriage tax is a necessary result of deviations from DB-ICR structure in the structure of tax rates (as occurred in 1969) and later in the structure of tax brackets (as in current law). Any policy that succeeds in achieving local marriage neutrality, equal treatment of married couples and a general marriage bonus must in one way or another reduce these deviations.

Certain extensions of these results seem worth exploring. Our results provide a starting point for examining tax structures that satisfy local marriage neutrality, equal treatment, and progressivity. It would be straightforward to model other policy proposals, like exempting some income from the lower–earning spouse,

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[23] That is to say, if $b_i < b_{i-1}$ yet $b_i > a_{i-1} + b_{i-1} z_{i-1} / z_{i-1}$, where $a_{i-1} < 0$.

in our framework. We could expand the number of categories beyond single individuals and married couples in order to model the tax code in greater detail. It may be fruitful to explore local versions of some of the other properties or to quantify these properties and model the tradeoffs more continuously. It has been suggested to us to consider intertemporal principles like neutrality with respect to marriage, divorce, and remarriage. While it is not clear which of these directions is most promising, there is in general great value for discovery, communication, and policy in creating these kinds of formal results.

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APPENDIX

The following preliminary results follow
immediately from Theorem 3.1.1, Corollary
3.1.9, and Remark 3.1.11 in Eichhorn (1978).

Let $y \in R_+$ and $y' \in R_+$. Let the three func-
tions $F_j: R_+ \rightarrow R, j = 1, 2, 3$ satisfy the functional
equation:

$$F_1(y + y') = F_2(y) + F_3(y')$$

Then there exist two real constants a and c and
a function $F: R_+ \rightarrow R$ such that $F_1(y) = F(y) + a + c$, $F_2(y) = F(y) + a$, $F_3(y) = F(y) + c$, and:

$$F(y + y') = F(y) + F(y').$$

Suppose further that F is continuous at one or
more points. Then:

$$F(y) = by$$

where b is a real constant.

Proof of Theorem 2

a. Fix any $y \geq 0$ and $y' \geq 0$. Marriage neutrality
and equal treatment give equations [1] and
[2]. Define $F_1(y + y') = T^s(y + y')$, $F_2(y) = T^s(y)$,
and $F_3(y') = T^s(y') - T^s(0)$. Then $F_1(y + y') =
F_2(y) + F_3(y')$ by equation [2]. It then follows
from the preliminary results that there ex-
ists a real number a and a function $F: R_+ \rightarrow
R$ with the property $F(y + y') = F(y) + F(y')$
such that $F_2(y) = F(y) + a$. We also know
that F is continuous at one or more points,
since by construction $F(y) = F_2(y) - a = T^s(y)
- a$ and by assumption T^s is continuous at
one or more points. Therefore (see the pre-
liminary results) there exists a real number
b such that $F(y) = by$. Substituting this into
the previous equation gives $T^s(y) = by + a$
and it follows that $a = T^s(0)$. Equation [1]
gives $T^m(y, y') = T^s(y + y') + T^s(0)$, so using
the previous result gives $T^m(y, y') = 2a + b(y
+ y')$. We have therefore established [4] and
[5]. The converse is straightforward. If
we add progressivity to the premises then
equations [1] – [3] hold for any $y > 0$ and
$y' > 0$. Equation [2] gives $T^s(y) + T^s(y') =
T^s(y + y') + T^s(0)$ and [3] then implies $T^s(0) <
0$. Equation [4] implies $a = T^s(0)$ so we con-
clude $a < 0$. Again, the converse is straight-
forward.

b. Marriage neutrality and equal payments
imply:

$$T^s(y + y') = T^m(y, y') = T^s(y) + T^s(y')$$

By assumption T^s is continuous at one or
more points, so it follows immediately (see
the preliminary results) that $T^s(y) = by$. The
converse is straightforward.

Proof of Theorem 3

a. Fix any interval i and any $y, y' \in [z_{i-1}, z_i]$. We
can follow the same steps as in the proof of
Theorem 2 to show that local marriage neu-

trality and equal treatment imply that there exist real numbers a_{i-1} and b_{i-1} such that the tax liability of a single individual with income y is $a_{i-1} + b_{i-1}y$ and the tax liability of a married couple with individual incomes y and y' is $2a_{i-1} + b_{i-1}(y + y')$. Notice that under the premise we have $y + y' \in [2z_{i-1}, 2z_i]$. It then follows from equal treatment of married couples that any couple with individual incomes such that $y + y' \in [2z_{i-1}, 2z_i]$ has tax liability $2a_{i-1} + b_{i-1}(y + y')$. Finally, since tax schedules can take only one value at each y (they are functions), it follows that for all i we must have $a_{i-1} + b_{i-1}z_i = a_i + b_iz_i$. The tax schedules are therefore piecewise linear and continuous. The converse is straightforward.

b. Suppose we add to the premise that the tax system contains a general marriage bonus. By the previous results we know that the tax system has the form in (a). The existence of a general marriage bonus implies there must be at least two distinct intervals, otherwise marriage neutrality would hold. Without any loss of generality we assume that all intervals are distinct, meaning for all i, $(a_{i-1}, b_{i-1}) \neq (a_i, b_i)$. Suppose there exists some i such that $b_{i-1} \geq b_i$. If $b_{i-1} = b_i$ then $a_{i-1} = a_i$ since $a_{i-1} + b_{i-1}z_i = a_i + b_iz_i$, and this contradicts distinctness. Therefore $b_{i-1} > b_i$. Fix incomes $y \in [z_{i-1}, z_i]$ and $y' \in [z_i, z_{i+1}]$ distinct from z_i such that $(y + y')/2 = z_i$. Then a general marriage bonus implies $2a_{i-1} + b_{i-1}(y + y') \leq a_{i-1} + b_{i-1}y + a_i + b_iy'$ and $2a_i + b_i(y + y') \leq a_{i-1} + b_{i-1}y + a_i + b_iy'$. Clearing both inequalities, combining and rearranging gives $y(b_i - b_{i-1}) \leq y'(b_i - b_{i-1})$. Since $0 \leq y < y'$ we must have $b_i - b_{i-1} \geq 0$, so $b_{i-1} \leq b_i$, a contradiction.

The only difficulty with the converse is showing that the tax system contains a general marriage bonus. We first show that T^s satisfies income splitting. We then give a lemma about the convexity of T^s that we use a number of times below. The convexity result and income splitting readily imply a general marriage bonus.

Assume that the tax system has the form in (a). Fix any nonnegative y and y'. There is some interval i such that $y + y' \in [2z_{i-1},$ $2z_i]$, so $2z_{i-1} \leq y + y' \leq 2z_i$ and so $(y + y')/2 \in [z_{i-1}, z_i]$. Therefore $T^m(y, y') = 2a_{i-1} + b_{i-1}(y + y') = 2[a_{i-1} + b_{i-1}(y + y')/2] = 2T^s[(y + y')/2]$. T^s satisfies income splitting.

The "convexity lemma" is as follows. Assume that the tax system has the form in (a) and $b_{i-1} < b_i$ for all $i \geq 1$. Fix any two intervals m and p with $m \leq p$, any two points $y_m \in [z_{m-1}, z_m]$ and $y_p \in [z_{p-1}, z_p]$, and any $\lambda \in [0, 1]$. Then:

[6] $T^s[\lambda y_m + (1 - \lambda)y_p] \leq \lambda T^s(y_m) + (1 - \lambda)T^s(y_p)$.

Furthermore, the inequality is strict if $\lambda \in (0, 1)$, $m < p$, and $y_m < z_m < y_p$.

To prove the lemma, we first show that, given any income, the actual tax is greater than or equal to the tax that would be computed for this income by extending any of the other pieces that define T^s. Formally, on any interval j the tax is given by $f_j(y) \equiv a_{j-1} + b_{j-1}y$. Fix any income $y_j \in [z_{j-1}, z_j]$. If there is an interval $j - 1$ then $z_{j-1} \leq y_j \Leftrightarrow (b_{j-1} - b_{j-2})z_{j-1} \leq (b_{j-1} - b_{j-2})y_{j-1} \Leftrightarrow a_{j-2} - a_{j-1} \leq (b_{j-1} - b_{j-2})y_j \Leftrightarrow f_{j-1}(y_j) \leq f_j(y_j)$. The two are equal if and only if $z_{j-1} = y_j$ (y_j is the lower bound of the jth interval). Repeating this for all $i \leq j - 1$ gives $f_i(y_j) \geq f_j(y_j)$ with equality if and only if $i = j - 1$ and $y_j = z_{j-1}$. A similar analysis establishes that for all $i \geq j + 1$ we have $f_i(y_j) \geq f_j(y_j)$ with equality if and only if $i = j + 1$ and $y_j = z_j$. Equality also holds if $i = j$.

Now fix any $y_m \in [z_{m-1}, z_m]$ and $y_p \in [z_{p-1}, z_p]$. Fix any $\lambda \in [0, 1]$. The convex combination of the incomes lies in some interval n. Necessarily $m \leq n \leq p$. The previous result gives $f_n(y_m) \leq f_m(y_m)$ and $f_n(y_p) \leq f_p(y_p)$. Therefore $T^s[\lambda y_m + (1 - \lambda)y_p] = f_n[\lambda y_m + (1 - \lambda)y_p] = \lambda f_n(y_m) + (1 - \lambda)f_n(y_p) \leq \lambda f_m(y_m) + (1 - \lambda)f_p(y_p) = \lambda T^s(y_m) + (1 - \lambda)T^s(y_p)$. T^s is convex.

Now assume $\lambda \in (0, 1)$, $m < p$, and $y_m < z_m < y_p$. Equation [6] holds with equality if and only if $f_n(y_m) = f_m(y_m)$ and $f_n(y_p) = f_p(y_p)$. $f_n(y_m) = f_m(y_m)$ if and only if ($n = m - 1$ and $y_m = z_{m-1}$), or ($n = m + 1$ and $y_m = z_m$), or $n = m$. The first contradicts $m \leq n \leq p$ and the second contradicts $y_m < z_m$, so $n = m$ is necessary for equality in [6]. $f_n(y_p) = f_p(y_p)$ if and only if ($n = p - 1$ and $y_p = z_{p-1}$), or ($n = p + 1$ and $y_p = z_p$),

or $n = p$. Using $n = m$ in the first gives $m = p - 1$ and $y_p = z_m$, contradicting $z_m < y_p$. Using $n = m$ in the third gives $m = n = p$, contradicting $m < p$. The second contradicts $m \le n \le p$. It follows that equality can not hold in [6]. This concludes the proof of the convexity lemma.

Keeping the notation above, fix any incomes y_m and y_p. Income splitting and convexity give:

[7] $\quad T^m(y_m, y_p) = 2T^s \left(\dfrac{y_m + y_p}{2} \right) \le T^s(y_m) + T^s(y_p).$

We have $\lambda = .5$, so the inequality is strict as long as $m < p$ and $y_m < z_m < y_p$. The tax system contains a general marriage bonus.

c. (First part). We established income splitting and the general marriage bonus in the proof of Theorem 3(b), so all we need to show is that the tax system favors married couples over singles. Fix any $y \ge 0$ and $y' \ge 0$. We have $a_0 \le 0$ so $T^s(0) \le 0$. If both y and y' are zero then $T^s(y) + T^s(y') = 2T^s(0) \le T^s(0) = T^s(y + y')$. Now suppose one of the two is positive. Define $\lambda = y/y + y'$. Equation [6] gives $T^s(y) = T^s[\lambda(y + y')] \le \lambda T^s(y + y')$ and $T^s(y') =$

$T^s[(1 - \lambda)(y + y')] \le (1 - \lambda) T^s(y + y')$. Adding the two gives:

[8] $\quad T^s(y) + T^s(y') \le T^s(y + y').$

This with [7] gives $T^m(y, y') \le T^s(y + y')$ for all incomes with strict inequality for some.

(Second part). T^s has the form in (a) and $b_{i-1} < b_i$ for all $i \ge 1$, so the convexity lemma applies. This with $a_0 \le 0$ (so $T^s(0) \le 0$) gives [8] for all incomes, and from the lemma the inequality is strict if both incomes are positive (so $\lambda \in (0, 1)$) and $z_1 < y + y'$. Equal payments then gives $T^m(y, y') = T^s(y + y') \ge T^s(y) + T^s(y')$ for all incomes with strict inequality for some. There is a general marriage tax.

We also prove the claim in footnote 9. Assume $y > 0$ and $y' > 0$. If $a_0 < 0$ then $T^s(0) < 0$. Then by [6]:

$$T^s\left[\left(\dfrac{y}{y + y'} \right) (y + y') \right] \le \left(\dfrac{y}{y + y'} \right) T^s(y + y')$$

$$+ \left(1 - \dfrac{y}{y + y'} \right) T^s(0) \; < \left(\dfrac{y}{y + y'} \right) T^s(y + y')$$

Clearing terms gives progressivity.