Lecture 19

1. Myers (1990)
2. Zodrow and Mieszkowski (1986), Part I.

1. Myers (1990)

   (a) Recall the relevant structure from the two-tax model (last lecture).

   Individuals receive wage income and income from owning and equal share of land in both regions. There is a head tax and a source based unit tax on land.

   Rents in region i are:

   $$R_i = f_i - n_i F_i$$

   An individual in region i has the budget constraint:

   $$x_i = w_i - \tau_{in} + \sum_k \left(\frac{R_k}{N} - \tau_{kr} \frac{T_k}{N} \right)$$

   Assuming $C_i(n_i, Z_i) = Z_i$, the budget constraint for the government in region i is:

   $$Z_i = \tau_{in} n_i + \tau_{ir} T_i$$

   (b) In this formulation, the property rights to land define a resource transfer between regions. The property tax modifies that transfer.

   This is how Myers wants you think about what is going on. He wants, however, to talk explicitly about the resource flow between regions. This leads him to rewrite the model.

   In the new formulation, the inter-regional transfer that would occur under the property tax is made an explicit choice variable for each regional government. The net land rent paid from region i to residents of j due to their ownership of land in i is:

   $$n_j \frac{R_i - \tau_{ir} T_i}{N}$$

   Myers defines:

   $$S_{ij} \equiv n_j \frac{R_i - \tau_{ir} T_i}{N}$$

   He then eliminates τ_{ir} from all expressions and replaces it with the expression in S_{ij}. Similarly for S_{ji} and τ_{jr}.

(c) The game he analyzes is played explicitly on transfer payments and local public goods.

The transfer payment implies a level for the property tax through the formula above. The property tax with the amount of public good and the government budget constraint implies a level for the head tax.

(d) Substituting S_{ij} and S_{ji} into the individual's budget constraint, using the definition of R_i and $w_i = F_i$ to eliminate w_i, and using the government's budget constraint to eliminate τ_{in} gives (do it!):

$$\begin{aligned} x_i &= \frac{f_i - R_i}{n_i} - \frac{Z_i - \tau_{ir} T_i}{n_i} + \frac{S_{ij}}{n_j} + \frac{S_{ji}}{n_i} \\ &= \frac{1}{n_i}\left[f_i(n_i, T_i) - Z_i + S_{ji} - S_{ij}\right] \\ &\equiv x_i(Z_i, n_i, S_{ij}, S_{ji}) \end{aligned}$$

(e) Migration equilibrium

The "utility hills" for regions i and j are, respectively:

$$U[x_i(Z_i, n_i, S_{ij}, S_{ji}), Z_i], \;\; U[x_j(Z_j, N - n_i, S_{ji}, S_{ij}), Z_j] \tag{1}$$

(I'm staying with the notation in the paper here).

Equalizing regional utility gives:

$$U[x_i(Z_i, n_i, S_{ij}, S_{ji}), Z_i] = U[x_j(Z_j, N - n_i, S_{ji}, S_{ij}), Z_j] \tag{2}$$

This implicitly defines the equilibrium population in i as a function of the public goods and transfers:

$$n_i = n_i(Z_i, Z_j, S_{ij}, S_{ji})$$

(f) For future reference, we define:

$$\begin{aligned} \mathcal{U}(Z_i, S_{ij}, Z_j, S_{ji}) &= U\left\{x_i[Z_i, n_i(.), S_{ij}, S_{ji}], Z_i\right\} \\ &= U\left\{x_j[Z_j, n_j(.), S_{ji}, S_{ij}], Z_j\right\} \end{aligned} \tag{3}$$

where:

$$x_i(Z_i, n_i, S_{ij}, S_{ji}) = \frac{1}{n_i}\left[f_i(n_i, T_i) - Z_i + S_{ji} - S_{ij}\right] \tag{4}$$

We also define:

$$U_i = U\left\{x_i[Z_i, n_i(.), S_{ij}, S_{ji}], Z_i\right\}$$

$$U_j = U\left\{x_j[Z_j, n_j(.), S_{ji}, S_{ij}], Z_j\right\}$$

(g) Nash equilibrium: Two possible one-shot games

i. Fiscal variables and population are determined simultaneously. This is *not* what Myers has in mind.

In this case, an equilibrium is a list:

$$Z_1, S_{12}, Z_2, S_{21}, n_1, n_2$$

such that each regional government maximizes the utility of its residents treating all variables other than its controls as constants and no individual wants to migrate.

This is easy to analyze. However, migration is not (in some sense) "anticipated."

ii. Alternatively, a strategy for regional government i is a choice of (Z_i, S_{ij}). Both regional governments move simultaneously.

Payoffs to each region are given by (3). In this formulation, $n_i(.)$ is an "assignment" of individuals to regions. This assignment rule is part of the rules of the game. It is common knowledge and we require it to assure that no individual has an incentive to migrate (it satisfies (2)).

A Nash equilibrium is a choice of fiscal variables such that each region is playing a best reply.

This is what Myers has in mind.

(h) Before turning to the technical aspects of the optimization problem, it is useful to get a sense of the interplay between these fiscal variables, the incentive to migrate and the utility of agents.

Figure 1 graphs the utility hills for each region (recall (1)). We are interested in how payoffs, meaning the equilibrium level of utility, and equilibrium population change with S_{ij} and S_{ji}.

Figure 1

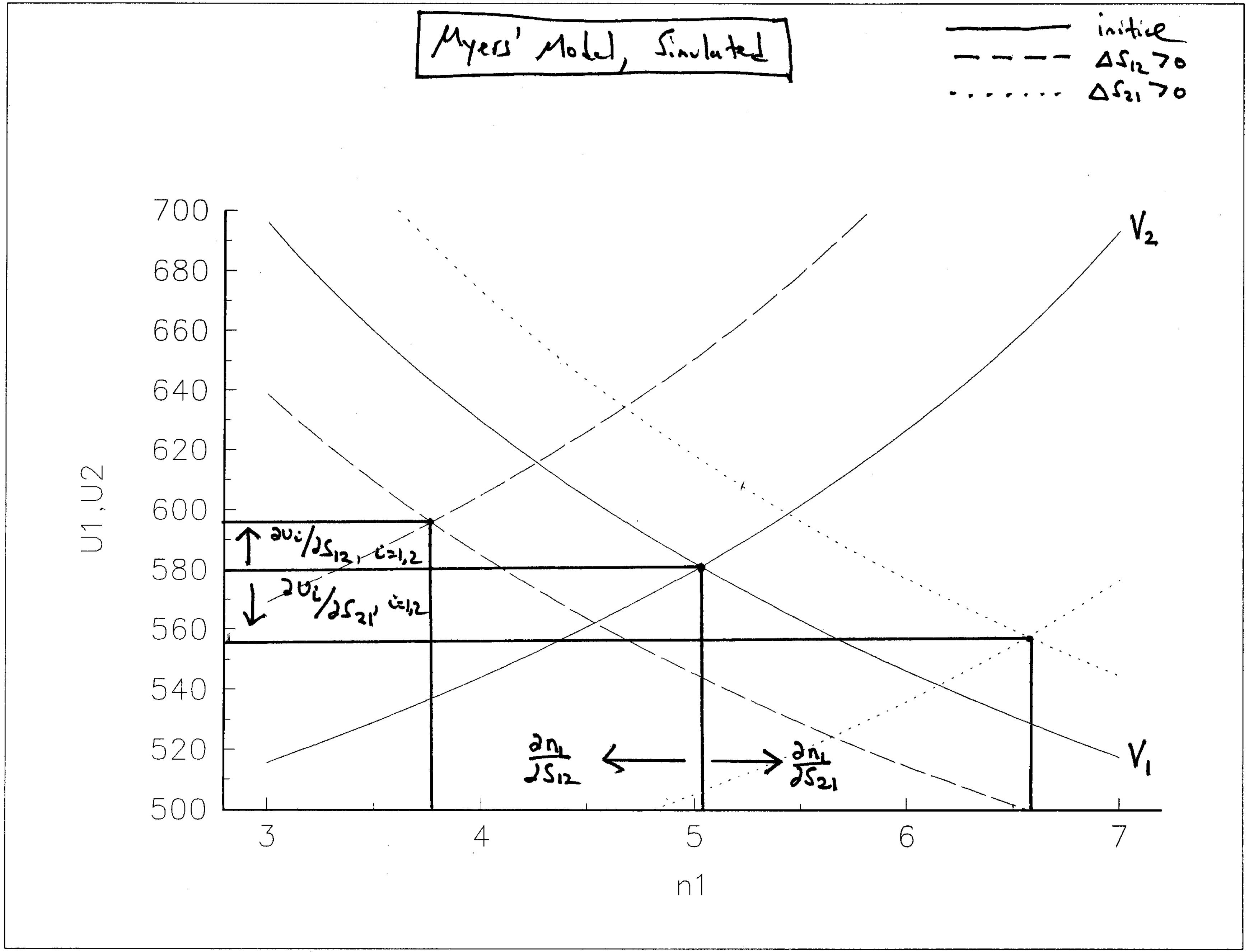
We have:

$$\frac{\partial \mathcal{U}}{\partial S_{ij}} = \frac{\partial U_i}{\partial S_{ij}} = \frac{\partial U_j}{\partial S_{ij}} = -\frac{\partial U_j}{\partial S_{ji}} = -\frac{\partial \mathcal{U}}{\partial S_{ji}}$$

What this is really saying is that only net transfers, $S_{ij} - S_{ji}$, affect the common level of utility.

Obviously, you can increase the net transfer from i to j by either increasing S_{ij} or reducing S_{ji}. The proposition says that both have the same effect on the common level of utility.

This holds because S_{ij} and S_{ji} enter both x_i and x_j symmetrically. Formally, the first and second equalities come from (3) and the definition of U_i and U_j. The formal proof of the third equality is below.



Myers' Model, Simulated
initial
ΔS12 > 0
ΔS21 > 0
U1, U2
700
680
660
640
620
600
580
560
540
520
500
3
4
5
6
7
n1
V2
V1
∂Ui/∂S12, i=1,2
∂Ui/∂S21, i=1,2
∂n1/∂S12
∂n1/∂S21

The proposition makes sense, but it is not entirely trivial.[1]

For population shifts:

$$\frac{\partial n_i}{\partial S_{ij}} = -\frac{\partial n_j}{\partial S_{ij}} = \frac{\partial n_j}{\partial S_{ji}}$$

We actually need both of these and derive them below.

(i) Regional government optimization

The regional government in i chooses:

$$Z_i, S_{ij}$$

to maximize

$$U_i = U\left\{x_i[Z_i, n_i(.), S_{ij}, S_{ji}], Z_i\right\} \tag{5}$$

subject to

$$S_{ij} \geq 0, Z_i \geq 0, \quad i = (1, 2), \quad j = (2, 1)$$

where $n_i(.)$ satisfies (2) and S_{ji} and Z_j are treated as constants.

(j) Derivatives.

Define and then derive (using (4)):

$$\begin{aligned} U_{ini} &\equiv U_{i1}\frac{\partial x_i}{\partial n_i} \\ &= U_{i1}\left(-\frac{x_i}{n_i} + \frac{F_i}{n_i}\right) \\ &= U_{i1}\frac{(F_i - x_i)}{n_i} \end{aligned} \tag{6}$$

i. The derivative of (5) with S_{ij} is:

$$\begin{aligned} \frac{\partial U_i}{\partial S_{ij}} &= U_{i1}\left(\frac{\partial x_i}{\partial S_{ij}} + \frac{\partial x_i}{\partial n_i}\frac{\partial n_i}{\partial S_{ij}}\right) \\ &= U_{i1}\frac{\partial x_i}{\partial S_{ij}} + U_{i1}\frac{\partial x_i}{\partial n_i}\frac{\partial n_i}{\partial S_{ij}} \\ &= -\frac{U_{i1}}{n_i} + U_{ini}\frac{\partial n_i}{\partial S_{ij}} \end{aligned} \tag{7}$$

[1] A trivial proposition is, if an increase in (gross) transfer from i to j raises the common level of utility, then a decrease in (gross) transfer from i to j must reduce the common level of utility, and by the same amount.

ii. The derivative of (5) with Z_i is:

$$\begin{aligned}\frac{\partial U_i}{\partial Z_i} &= U_{i1}\left(\frac{\partial x_i}{\partial Z_i} + \frac{\partial x_i}{\partial n_i}\frac{\partial n_i}{\partial Z_i}\right) + U_{i2} \\ &= U_{i2} - \frac{U_{i1}}{n_i} + U_{ini}\frac{\partial n_i}{\partial Z_i} \qquad (8)\end{aligned}$$

(k) We need to evaluate $\frac{\partial n_i}{\partial S_{ij}}$ and $\frac{\partial n_i}{\partial Z_i}$.

These come from (2) and the implicit function theorem. Define:

$$\phi(.) = U\left[x_i(Z_i, n_i, S_{ij}, S_{ji}), Z_i\right] - U\left[x_j(Z_j, N - n_i, S_{ij}, S_{ji}), Z_j\right]$$

i. The effect of transfers on population:

$$\frac{\partial n_i}{\partial S_{ij}} = -\frac{\partial \phi/\partial S_{ij}}{\partial \phi/\partial n_i} = \frac{U_{i1}/n_i + U_{j1}/n_j}{U_{ini} + U_{jnj}} \qquad (9)$$

Switching i and j on the right hand side gives the same expression. Immediately then we also know:

$$\frac{\partial n_i}{\partial S_{ij}} = \frac{\partial n_j}{\partial S_{ji}} \qquad (10)$$

Also, using (2), the definition of x_j and the previous results gives:

$$\begin{aligned}\frac{\partial U_i}{\partial S_{ij}} &= \frac{\partial U_j}{\partial S_{ij}} \\ &= U_{j1}\left(\frac{\partial x_j}{\partial S_{ij}} + \frac{\partial x_j}{\partial n_j}\frac{\partial n_j}{\partial S_{ij}}\right) \\ &= U_{j1}\left(-\frac{\partial x_j}{\partial S_{ji}} - \frac{\partial x_j}{\partial n_j}\frac{\partial n_j}{\partial S_{ji}}\right) \\ &= -\frac{\partial U_j}{\partial S_{ji}} \qquad (11)\end{aligned}$$

ii. The effect of local public good on population:

$$\frac{\partial n_i}{\partial Z_i} = -\frac{\partial \phi/\partial Z_i}{\partial \phi/\partial n_i} = \frac{U_{i1}/n_i - U_{i2}}{U_{ini} + U_{jnj}} \qquad (12)$$

(l) We can now express the derivatives in the form in which they are most useful.

i. Using (6) in (7) gives:

$$\frac{\partial U_i}{\partial S_{ij}} = \frac{U_{i1}}{n_i}\left[-1 + (F_i - x_i)\frac{\partial n_i}{\partial S_{ij}}\right] \qquad (13)$$

ii. Using (12) in (8) gives:

$$\begin{aligned}\frac{\partial U_i}{\partial Z_i} &= U_{i2} - \frac{U_{i1}}{n_i} + U_{ini}\frac{U_{i1}/n_i - U_{i2}}{U_{ini} + U_{jnj}} \\ &= U_{i2} - \frac{U_{i1}}{n_i} + (U_{i1}/n_i - U_{i2})\frac{U_{ini}}{U_{ini} + U_{jnj}}\end{aligned}$$

$$= \left(U_{i2} - \frac{U_{i1}}{n_i}\right)\left(1 - \frac{U_{ini}}{U_{ini} + U_{jnj}}\right) \tag{14}$$

(m) Now consider the Kuhn-Tucker conditions from the governments' optimization problems.

First, we have:

$$\frac{\partial U_i}{\partial S_{ij}} \leq 0, \quad S_{ij} \geq 0, \quad S_{ij}\frac{\partial U_i}{\partial S_{ij}} = 0, \quad i = (1,2), \quad j = (2,1)$$

This with (11) implies that if one of the derivatives is strictly negative then the other is strictly positive, a contradiction. Therefore:

$$\frac{\partial U_i}{\partial S_{ij}} = \frac{\partial U_j}{\partial S_{ji}} = 0$$

Apply (13):

$$\frac{\partial U_i}{\partial S_{ij}} = \frac{U_{i1}}{n_i}\left[-1 + (F_i - x_i)\frac{\partial n_i}{\partial S_{ij}}\right] = 0$$

Use (10):

$$F_i - x_i = \frac{1}{\partial n_i / \partial S_{ij}} = \frac{1}{\partial n_j / \partial S_{ji}} = F_j - x_j \tag{15}$$

Thus, this necessary condition for Nash equilibrium gives one of the necessary conditions for Pareto Efficiency.

(n) We also have:

$$\frac{\partial U_i}{\partial Z_i} \leq 0, \quad Z_i \geq 0, \quad Z_i\frac{\partial U_i}{\partial Z_i} = 0, \quad i = 1,2$$

This with (14) gives:

$$\frac{\partial U_i}{\partial Z_i} = \left(U_{i2} - \frac{U_{i1}}{n_i}\right)\left(1 - \frac{U_{ini}}{U_{ini} + U_{jnj}}\right) \leq 0$$

If $U_{ini} = 0$ then the expression reduces to the first term in parentheses. If $U_{ini} \neq 0$ then using (6) and (15) give:

$$\begin{aligned} \frac{U_{ini}}{U_{ini} + U_{jnj}} &= \frac{U_{i1}\frac{(F_i - x_i)}{n_i}}{U_{i1}\frac{(F_i - x_i)}{n_i} + U_{j1}\frac{(F_j - x_j)}{n_j}} \\ &= \frac{1}{1 + \frac{U_{j1}/n_j}{U_{i1}/n_i}} \end{aligned}$$

This is strictly between zero and 1. In either case we have:

$$n_i\frac{U_{i2}}{U_{i1}} \leq 1$$

The usual Samuelson condition holds at interior solutions.

(o) Optimal net interregional transfer: Myers' equation (4.1).

We give a direct derivation of the interregional transfer that a central planner would make.

i. Define:

$$S = S_{12} - S_{21}$$

This is the net transfer from region i to region j. This could be negative if region 1 sends less to 2 than region 2 sends to 1.

ii. Regional resource constraints:

$$f_1(n_1, T_1) - S = x_1 n_1 + Z_1$$

$$f_2(n_2, T_2) + S = x_2 n_2 + Z_2$$

Solving for x_1 and x_2:

$$x_1 = \frac{f(n_1, T_1) - S - Z_1}{n_1}$$

$$x_2 = \frac{f(n_2, T_2) + S - Z_2}{n_2}$$

The Pareto problem now is for the central planner to choose

$$Z_1, Z_2, S, n_1, n_2$$

to find a stationary point of:

$$\begin{aligned}
\mathcal{L} &= U\left[\frac{f_1(n_1, T_1) - S - Z_1}{n_1}, Z_1\right] \\
&+ \lambda\left\{U\left[\frac{f_2(n_2, T_2) + S - Z_2}{n_2}, Z_2\right] - \bar{U}\right\} \\
&+ \psi[N - n_1 - n_2]
\end{aligned}$$

iii. First order conditions (in the order above):

$$U_{12} - \frac{U_{11}}{n_1} = 0$$

$$\lambda\left(U_{22} - \frac{U_{21}}{n_2}\right) = 0$$

$$-\frac{U_{11}}{n_1} + \lambda\frac{U_{21}}{n_2} = 0$$

$$\frac{U_{11}}{n_1}\left[\frac{n_1 F_1 - f_1(n_1, T_1) + S + Z_1}{n_1}\right] - \psi = 0$$

$$\lambda\frac{U_{21}}{n_2}\left[\frac{n_2 F_2 - f_2(n_2, T_2) - S + Z_2}{n_2}\right] - \psi = 0$$

iv. The first two give the Samuelson conditions.

v. The last three give:

$$\frac{n_1F_1 - f_1(n_1,T_1) + S + Z_1}{n_1} = \frac{n_2F_2 - f_2(n_2,T_2) - S + Z_2}{n_2}$$

Rearranging gives:

$$S = \frac{n_1 n_2}{N}\left[\frac{f_1(n_1,T_1) - n_1F_1}{n_1} - \frac{f_2(n_2,T_2) - n_2F_2}{n_2} + \frac{Z_2}{n_2} - \frac{Z_1}{n_1}\right]$$

vi. This is a famous formula. You will see it in some of the papers in this literature.

(p) Important technical point.

As a technical matter, what has been shown is that the first order conditions for the equilibrium problem are the same as the first order conditions for the optimum problem. Strictly speaking, this does not establish a first welfare theorem. We do not know that every equilibrium is an optimum.

Figure 2

2. Zodrow and Mieszkowski, Part I.

Regarding the underprovision of residential public services:

A tax on mobile capital to finance local public goods leads to an inefficient allocation.

They show that the Samuelson condition does not hold.

They *also* show that the public good is underprovided, in the sense that the equilibrium quantity is less than the equilibrium quantity when communities can use a head tax. The latter allocation is both Pareto improving and efficient, so it is fair to say that the tax on capital leads to underprovision.

Contrast with Atkinson and Stern.

Regarding the underprovision of business public services:

A tax on mobile capital to finance local public infrastructure leads to an inefficient allocation.

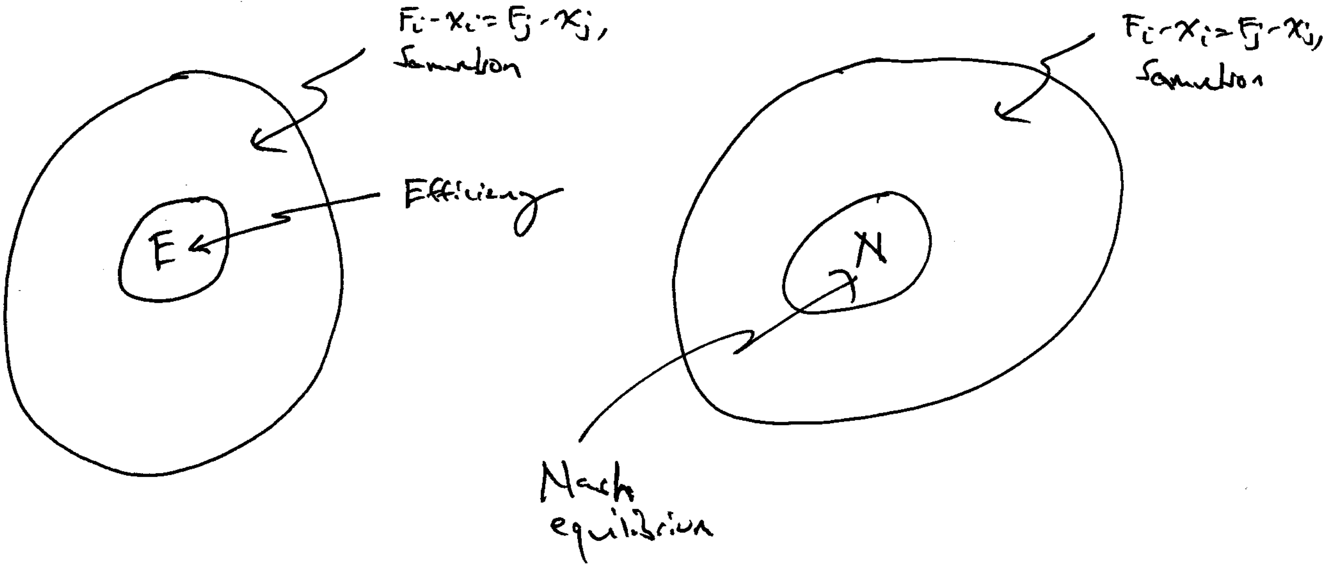
Here we examine the underprovision of residential public services.

(a) The national economy consists of N small and identical jurisdictions.

Each has the same amount of land.

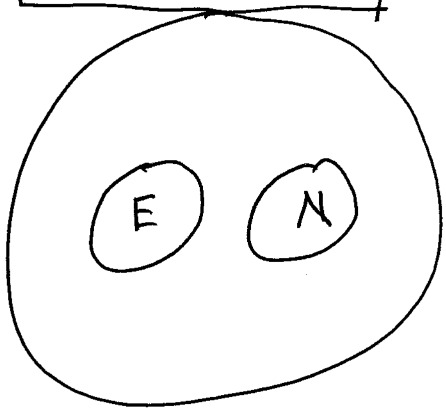
Each has the same number of identical and immobile residents.

(b) The population of each community is normalized to 1. Therefore, N is both total population and total number of communities.

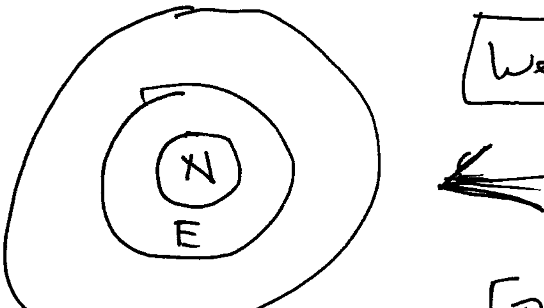


So:

We could have:



We'd like to have



[This would follow if the two conditions were sufficient for efficiency.]

(c) The nation has a fixed stock of capital $\bar{K}$.

The capital in any jurisdiction is denoted K.

Capital is perfectly mobile across jurisdictions. Thus, it must earn the same net return in every jurisdiction.

The *net* return is denoted r.

(d) Individuals derive utility from consumption C and publicly provided private good P:

$$U(C, P)$$

(e) "Output" (all purpose good) is produced by competitive firms within each jurisdiction using land and capital.

$$F(K), \quad F_K > 0, \quad F_{KK} < 0$$

Output can be transformed (globally) into C and P in a 1:1 ratio. So, $MRT_{CP} = 1$.

(f) Producers treat r as exogenous.

At the profit maximizing level of production, the quantity of capital employed is such that its marginal product equals its gross price:

$$r + T = F_K(K)$$

where T is a unit tax on capital.

This determines the demand for capital as a function of the net return and tax:

$$K(r + T) \tag{16}$$

(g) Denote the derivative:

$$K'$$

For the derivative with T, define:

$$\theta \equiv r + T - F_K(K)$$

Then:

$$\frac{\mathrm{d}K}{\mathrm{d}T} = K' = -\frac{\partial\theta/\partial T}{\partial\theta/\partial K} = -\frac{1}{-F_{KK}} = \frac{1}{F_{KK}} < 0$$

Capital taxes drive out capital.

(h) Local governments fund the publicly provided good (P) with the tax on capital (T).

There may also be an exogenous head tax levied at the same rate in each community (H).

Therefore the community's budget constraint is:

$$P = TK + H$$

(i) Each individual owns an equal share of the land in the jurisdiction of residence and an equal share of the national capital stock.

The individual budget constraint is therefore:

$$C = F(K) - (r+T)K + r(\bar{K}/N) - H$$

This is per-capita land rents plus the per-capita share of the total return to capital less the head tax.

(j) Substituting the individual and government budget constraints into the utility function and $K(r+T)$ for K gives "indirect utility" V. Doing this in steps, define:

$$C(H,T) \equiv F[K(r+T)] - (r+T)K(r+T) + r(\bar{K}/N) - H$$

$$P(H,T) \equiv TK(r+T) + H$$

So:

$$V(H,T) = U[C(H,T), P(H,T)] =$$

$$\equiv U\{F[K(r+T)] - (r+T)K(r+T) + r(\bar{K}/N) - H, TK(r+T) + H\}$$

(k) Each local government treats r as exogenous (just as the producers do).

The local government does, however, recognize the dependence of K on T through (16). That is to say, it recognizes that the tax on capital will affect the amount of capital in the region.

The local government chooses T and perhaps H to maximize V above.

Note that the governments are assumed to play the game in tax rates rather than public goods.

Also, the government moves first, and then individuals are left to consume private good from their net incomes.

(l) The first order condition with T is:

$$\frac{\partial V}{\partial T}(H,T)$$

$$= (U_C)[F_K K' - (r+T)K' - K] + (U_P)(K + TK')$$

$$= 0$$

Using $r + T = F_K$, this reduces to:

$$(U_C)(-K) + (U_P)(K + TK') = 0 \quad (17)$$

(m) Suppose we exogenously assume $H = 0$, so the only tax is the capital tax. Then we must have $T > 0$ to have any public good. Furthermore, we can rearrange (17) to give:

$$\frac{U_P}{U_C} = \frac{K}{K + TK'} > 1$$

where the inequality follows from $T > 0$ and $K' < 0$.

The Samuelson condition does not hold.

Note that the allocation of capital is not distorted, it is the same as it would be at an efficient allocation. The problem is that the common tax rate on capital is too low. If all regions could be forced to increase the tax rate on capital, the common level of utility would rise. This would not be an equilibrium, however – the force would have to remain!

(n) If both the head tax and the local property tax can be chosen by each region then we have (17) together with:

$$\frac{\partial V}{\partial H}(H, T) = -U_C + U_P = 0$$

Therefore:

$$\frac{U_P}{U_C} = 1$$

The only way this can hold with (17) is to have $T = 0$. In equilibrium, communities would use only the head tax. Furthermore, the Samuelson condition would hold.

(o) The authors then want to show that the public good is underprovided when communities have only the tax on capital.

To do this, they vary H from zero to its optimal value, allowing T to adjust optimally for given H and allowing r to adjust to clear the market for capital. This creates a path of equilibria. They show that the chosen level of public spending increases with H along the path.

(p) Formally, use the previous expressions (but making r explicit) and equation (17) to define:

$$\begin{aligned} F_1(H, T, r) \equiv\ & -U_C[C(H, T, r), P(H, T, r)]K(r + T) \\ & + U_P[C(H, T, r), P(H, T, r)][K(r + T) + TK'(r + T)] \\ & = 0 \end{aligned}$$

The market clearing condition for capital gives:

$$F_2(H, T, r) \equiv NK(r + T) - \bar{K} = 0$$

These simultaneously define the functions:

$$T(H), r(H)$$

What the authors want to show is that the following increases with H:

$$P^*(H) \equiv T(H)K[r(H)+T(H)]+H$$

We have:

$$\frac{\mathrm{d}P^*}{\mathrm{d}H} = \frac{\mathrm{d}T}{\mathrm{d}H}K + TK'\left(\frac{\mathrm{d}r}{\mathrm{d}H} + \frac{\mathrm{d}T}{\mathrm{d}H}\right) + 1$$

We can evaluate this using:

$$\frac{\mathrm{d}T}{\mathrm{d}H} = -\frac{\det\begin{pmatrix} \frac{\partial F_1}{\partial H} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial H} & \frac{\partial F_2}{\partial r}\end{pmatrix}}{\det\begin{pmatrix} \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial r}\end{pmatrix}}$$

$$\frac{\mathrm{d}r}{\mathrm{d}H} = -\frac{\det\begin{pmatrix} \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial H} \\ \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial H}\end{pmatrix}}{\det\begin{pmatrix} \frac{\partial F_1}{\partial T} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial T} & \frac{\partial F_2}{\partial r}\end{pmatrix}}$$

Notice that the numerator of the first expression is:

$$\frac{\partial F_1}{\partial H}\frac{\partial F_2}{\partial r} - \frac{\partial F_2}{\partial H}\frac{\partial F_1}{\partial r} = \frac{\partial F_1}{\partial H}K'$$

and the numerator of the second is:

$$\frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial H} - \frac{\partial F_2}{\partial T}\frac{\partial F_1}{\partial H} = -\frac{\partial F_1}{\partial H}K'$$

Thus, an increase in the head tax produces offsetting effects on the optimal choice of T and the equilibrium value of r:

$$0 = \left(\frac{\mathrm{d}r}{\mathrm{d}H} + \frac{\mathrm{d}T}{\mathrm{d}H}\right)$$

It now follows that:

$$\frac{\mathrm{d}P^*}{\mathrm{d}H} = \frac{\mathrm{d}T}{\mathrm{d}H}K + 1$$

It should be straightforward now to show that this is positive.